# Surprises of Phase Transition Astrophysics

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It is a half-century-long story about the first-order phase transition effects on equilibrium, stability and pulsations of planets and stars. The topics more or less considered in author's papers are mainly touched, and no attempt was done to cover all literature on subject.
It was Ramsey who in 1950 had first shown in MNRAS paper [1] that *if* in the center of a planet, with central pressure $P_c$ just reaching critical pressure $P_0$, the first-order phase transition takes place with density jump from $\rho_1$ to $\rho_2 = q * \rho_1$, and *if* $q > 1.5$, then planet loses its stability *at the same moment.* This remarkably amazing and quite unexpected result has been since then rediscovered by many authors repeatedly - last time in nucl-th/9902033.

*"...- D'you think all these (cloths) will be the wear?*
*- I think all these should be tailored."*
Yuri Levitanski, Soviet Jewish poet

The story began to me some 35 years ago when trying to understand paper [2], I've found that something is wrong with Mass-Central Density curves for white dwarf stars near maximum - there should be *sharp*, not *smooth* maximum of mass, as reverse $\beta$-decay reactions lead to *the discontinuity of density distribution* inside the star.

To simplify the problem, I used some models, allowing *analytical* investigation - polytropes with indices $n$ equal to 0 and 1 in envelope and core in various combinations. In all cases considered, it happened that *if* $q > 1.5$, the instability occurs at the moment the phase transition starts in the center of the star.

My supervisor acad. Ya.B. Zeldovich for a long time did not believe in such "*strange*" figure 3/2, so I kept analyzing various models. After a while Ya.B. became himself convinced and within a brief period he developed an amazingly elegant method of prooving this *word constant 3/2*, and a joint paper [4] was submitted to Astronom. Zhurnal. Needless to say, I was happy that I was right and that I had joint paper with such an outstanding scientist. Catastrophe came short: Ya.B. sent me a brief letter noting that *"the result 3/2 is known in literature!"*.

No comments...

I've never managed since then to write another joint paper with Ya.B....

Why 3/2? I do not know, but it seems that 3/2 *number* is directly related to the $r^{-1}$ - law of potential $U(r)$ in Newtonian Theory of Gravitation (NTG) in $3D$ space[11].

This is not the last surprise of Phase-Transition Astrophysics (PTA)!

In the classical theory [5] of equilibrium and stability of stars comprised of matter with *smooth* Equations of State (EoS), there is a common sense that *rotation leads to the increase of stability reserve* while *General Relativity (GR) leads to the decrease of stability.*

And you may guess that exactly *opposite* situation is in PTA. In GR, the critical value of *energy* density $q = \varepsilon_2/\varepsilon_1$ is [6]: $3/2\ (1 + P_0/\varepsilon_1)$, that is *larger* than in NTG.

Why *larger*? - I do not know...

As to rotation, it was found [7], that for steady-state rotation with *small* angular velocity $\Omega$, $q_{crit} = 3/2 - \Omega^2 / 4\pi G\rho_1$, that is, of course, rotation *reduces* the stability of star *against* phase-transition-induced instability.

Why *reduces*? - I do not know...

Abovementioned *three suprises* of PTA, combined in the formula

$$q_{crit} = 3/2 \; - \; \Omega^2 \; / \; 4\pi G\rho_1 \; + 3/2 \; (1 + P_0/\rho_1 c^2),$$

are valid for *any* EoS's of old and new phases.

A number of amazing results was found analysing various particular models. First to be mentioned is the two-constant-density-phase model with First-Order Phase Transition (PT1) at the boundary between core and envelope. This model was extensively used for different problems - general dependence of Mass-Radius etc. curves, effects of rotation and GR, neutral core (when PT1 *begins at some distance from the center of a star*) etc.

In the last case, it was found [3] that $q_{crit}$ is an increasing function of size of neutral core: $q_{crit} = 1 + k/[3 - 4\;x - (k-1)\;x^4]$, where $x$ is *relative* radius of neutral core, and $k$ is relation of density in neutral core to density in envelope; and $q_{crit} \to \infty$ at $x \;\to x_{cr}(k)$, where for example at $k = 1$, $x_{cr} = 3/4$ - another amazing value. At *larger* neutral cores, PT1 with arbitrary large $q$ can not force a star to lose its dignity and stability!

For a model with polytropic indices $n = 1$ both in envelope and neutral core [3], $x_{cr} =$ .6824 (corresponding relative mass of core is .6375).

Returning to $n = 0$, for $q > 3/2$ there is another critical point in $Mass - P_c$ curves, where at the minimum of mass the *recovery* of stability takes place, and for larger $P_c$ there is a branch of stable equilibrium states. At $the minimum \;\; of \;\; Mass$:

$$f(q, x) = (q-1)^2 x^4 \; + \; 4\;(q-1)\;x + (3-2q) = 0. \tag{1}$$

It was found [8] that GR effects, in $the\ first\ Post-Newtonian\ approximation$ lead to the PN-correction to $x$ in Eq. (1):

$$\Delta_{PN}(q, x) = \frac{9 - 7\;q + 27(q-1)x + (q-1)(4q-27)x^2 + (q-1)(9-4q)x^3}{2(q-1)[1+(q-1)x^3]^3}\frac{P_0}{\rho_1 c^2}. \tag{2}$$

The surprises have not finished - this correction is *negative* at small values of $q$ and *positive* at $q > 1.89$. That is for larger $q$, GR effect is of *correct* sign and *reduces* the region of stability at $(q - x)$ plane.

For the same $n = 0$ model, the analytical formula for frequency, $\omega$, of small adiabatic radial pulsations of the lowest mode can be found:

$$\omega_0^2 = \frac{4\pi\;G\;\rho_1\;f(q,x)}{3\;(q-1)(1-x)}, \tag{3}$$

and in the case of *slow* rotation with angular velocity $\Omega$ [10]:

$$\omega_\Omega^2 = \omega_0^2 + \Delta_\Omega(q, x), \quad \text{with}$$

$$\Delta_\Omega(q, x) = \frac{2}{3}\;\Omega^2 \left[\frac{5x(1-x)(1+x)^2}{1+(q-1)x^5} - \frac{1+(q-1)x}{(q-1)(1-x)}\right]. \tag{4}$$

Rotational correction to frequency squared is *negative* at smaller values of $x$, that is rotation reduces the stability reserve of star with PT1, then for larger value of $x$, rotational effect is of "*correct*" sign.

In fact, dependence of both corrections, due to GR and rotation, on $x$ and $q$ is rather complicated, and Fig. 1 presents only a part of $(q{-}x){-}$plane with lines on which $\Delta_\Omega(q,x) = 0$ (broken line labeled as "Rot")and $\Delta_{PN}(q,x) = 0$ (dash-point line labeled as "GR"). Also shown is the curve $f(q,x) = 0$ (solid line labeled as "Crit") from Eq.1, which marks a boundary between *stable* equilibrium states (right-hand) from *unstable* ones (left-hand). Remarkably, this curve crosses *both* curves of *zero correction.* Due to GR, curve $f(q,x) = 0$ is forced to rotate clockwise around point of intersection of lines "GR" and "Crit", while rotation makes the critical curve rotate counterclockwise around point of intersection of lines "Rot" and "Crit".

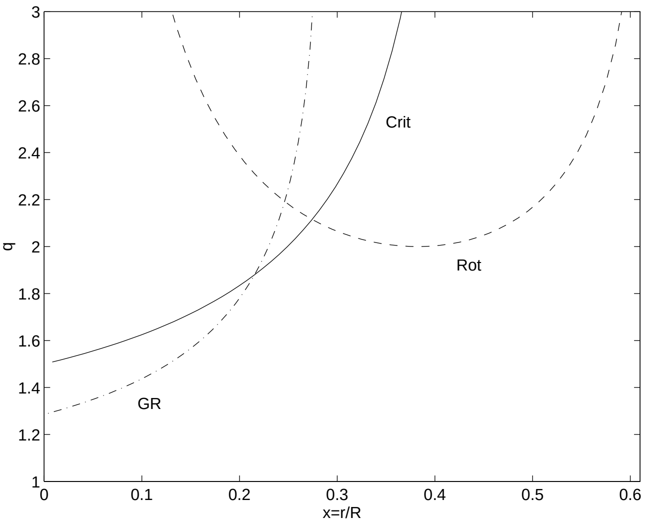


FIG. 1. Three important curves at $(q - x){-}$plane: critical state of star stability recovery, see Eq. 1, and GR and Rotational corrections equal to zero, see Eq. 2 and Eq. 4, respectively.

Why do rotation and GR exert such strange effects on stability of a star with PT1?

- *I do not know...*

A lot of interesting and unsolved problems of pulsations (eigen- values and functions, damping), strong rotation with regard to deviation of equilibrium figures from spherical symmetry, considering more EoS, etc. have been left aside , but 1500-word mark is close and I pass to the epilogue...

Most of this story happened to me many years ago, in the Soviet Union, the Power so unexpectedly crushed in the recent past, as if some first-order phase transition had acted in the center of It...

Now I'm in Israel, all my papers being left over there and these lines being written by heart, by memory, with no paper at hand...

And last sentences (hopefully still in 1500-word limit), returning to epigraph:

- Do I believe this all *will be* awarded?
- I think this all *should be* written.

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